Solutionbank S1
Edexcel AS and A Level Modular Mathematics

Examination style paper
Exercise A, Question 1

Question:

1 A fair die has six faces numbered 1, 1, 1, 2, 2 and 3. The die is rolled twice and the number showing on the uppermost face is recorded.

Find the probability that the sum of the two numbers is at least three. (5)

Solution:

3	4	4	4	5	5	6
2	3	3	3	4	4	5
2	3	3	3	4	4	5
1	2	2	2	3	3	4
1	2	2	2	3	3	4
1	2	2	2	3	3	4
Second First	**1**	**1**	**1**	**2**	**2**	**3**

The easiest solution involves drawing a diagram to represent the sample space. Each square is the sum of the scores on the die. The first method mark is for attempting the diagram and the second is an accuracy mark for all the values correct.

Each of the values that are 'at least 3' are underlined; 3, 4, 5 & 6.

M1A1A1

$$\text{P(Sum at least } 3) = \frac{27}{36} = \frac{3}{4}$$

There are 27 values underlined and 36 values in the sample space. Then cancel the fraction.
M1A1

ALTERNATIVE SOLUTION

Let D_1 = the number on the first die
and D_2 = the number on the second die

$$\begin{aligned} \text{P}(D_1 + D_2 \geq 3) &= 1 - \text{P}(D_1 + D_2 = 2) \\ &= 1 - \text{P}(D_1 = 1 \text{ and } D_2 = 1) \\ &= 1 - \text{P}(D_1 = 1) \times \text{P}(D_2 = 1) \\ &= 1 - \frac{1}{2} \times \frac{1}{2} \\ &= \frac{3}{4} \end{aligned}$$

This is a slightly quicker solution.

$\text{P}(D = 1) = 0.5$ and D_1 and D_2 are independent so the probabilities are multiplied together.

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Examination style paper
Exercise A, Question 2

Question:

Jars are filled with jam by a machine. Each jar states it contains 450 g of jam.

The actual weight of jam in each jar is normally distributed with mean 460 g and standard deviation of 10 g.

a Find the probability that a jar contains less then the stated weight of 450 g. (3)

Jars are sold in boxes of 30.

b Find the expected number of jars containing less than the stated weight. (2)

The standard deviation is still 10 g. The mean weight is changed so that 1% of the jars contain less than the stated weight of 450 g of jam.

c Find the new mean weight of jam. (4)

Solution:

(a)

$$P(X < 450) = P\left(Z < \frac{450-460}{10}\right) = P(Z < -1.0)$$

$$= 1 - 0.8413 = 0.1587$$

Standardise by subtracting the mean and dividing by the standard deviation gets the first method mark and the z value of -1.0 gets the accuracy mark.

(b)

Expected number of jars = 30×0.1587
$= 4.761$ or 4.76 or 4.8

(c)

$$P(X < 450) = 0.01$$

$$\frac{450-\mu}{10} = -2.3263$$

$$\mu = 473.263 = 473 \text{ to } 3 \text{ sf}$$

Forming the correct equation with the new mean as an unknown gets the method and accuracy mark, the B mark is awarded for getting −2.3263 from the tables

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Examination style paper
Exercise A, Question 3

Question:

A discrete random variable X has a probability distribution as shown in the table below

x	0	1	2	3
$\mathbf{P(X = x)}$	0.2	0.3	b	$2a$

where a and b are constants.

If E(X) = 1.6,

a show that $b = 0.2$ and find the value of a. (5)

Find

b $E(5 - 2X)$, (2)

c $\text{Var}(X)$, (3)

d $\text{Var}(5 - 2X)$. (2)

Solution:

(a)

$0.5 + b + 2a = 1$

$0.3 + 2b + 6a = 1.6$

Solving

$a = 0.15,\ b = 0.2$

Remember that adding all the probabilities together equals 1.

The second equation is formulated from the value of the expectation. Multiply the values of X by the associated probabilities and equate to 1.6.

(b)

$$\begin{aligned} E(5-2X) &= 5 - 2E(X) \\ &= 5 - 2 \times 1.6 = 1.8 \end{aligned}$$

(c)

$\text{Var}(X) \ = 1^2 \times 0.3 + 2^2 \times 0.2 + 3^2 \times 0.3 - 1.6^2$

$= 1.24$

For the variance you square each value of x and multiply by the probability. Remember to subtract the square of the expectation.

(d)

Var (5 − 2X) = 4 × Var(X) = 4.96

Solutionbank S1
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Examination style paper
Exercise A, Question 4

Question:

The attendance at college of a group of 20 students was recorded for a four week period. The numbers of students attending each of the 16 classes are shown below.

20	20	19	19
18	19	18	20
20	16	19	20
17	19	20	18

a Calculate the mean and the standard deviation of the attendance. (4)

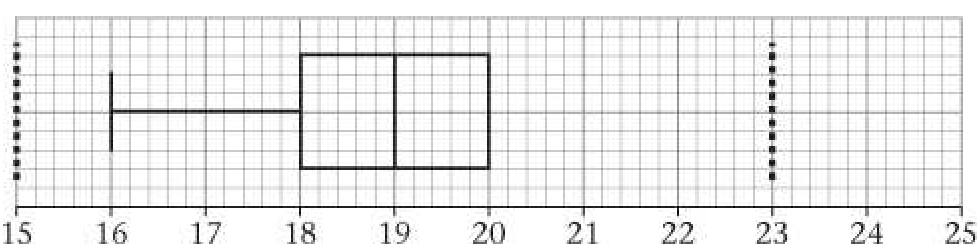
b Express the mean as a percentage of the 20 students in the group. (1)

In the same four week period, the attendance of a different group of 22 students was recorded.

22	18	20	21
17	16	16	17
20	17	18	19
18	20	17	16

c Find the mode, median and inter-quartile range for this group of students. (3)

A box plot for the first group of students is drawn below.



d Using the same scale draw a box plot for the second group of students. (3)

The mean percentage attendance and standard deviation for the second group of students are 82.95 and 1.82 respectively.

e Compare and contrast the attendance of each group of students. (3)

Solution:

(a)

$\bar{x} = \frac{302}{16} = 18.875$

standard deviation is $\sqrt{\frac{5722}{16} - 18.875^2} = \sqrt{1.359375}$

Set out your working clearly so you will still be given the method mark if you make a calculator error.

= 1.16592…

(b) mean % attendance is $\frac{18.875}{20} \times 100 = 94.375$

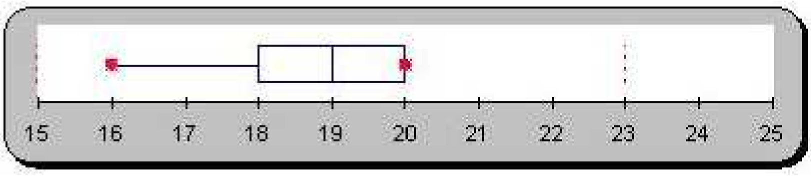
(c) Mode is 17

Median is 18

IQR is 20 − 17 = 3

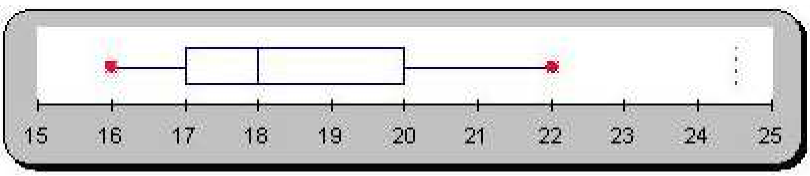
(d)

First Group:



Second Group:

Put the box plots side by side so you can compare easily.



(e)

First mean % > Second mean %

First IQR < Second IQR

First sd < Second sd

First range < Second range

First negative skew, given by whiskers, symmetric by box

Second positive skew.

There are 3 marks for this part, so 3 different correct comments are required. Try to comment about location, spread and shape.

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Exercise A, Question 5

Question:

The random variable X has the distribution

$\mathrm{P}(X = x) = \frac{1}{n}$ for $x = 1, 2, \ldots, n$.

a Write down the name of the distribution. (1)

Given that $\mathrm{E}(X) = 10$,

b show that $n = 19$, (2)

c Find $\mathrm{Var}(X)$. (2)

Solution:

(a) Discrete uniform distribution

(b)

$$\frac{(n+1)}{2} = 10$$
$$n = 19$$

Learning the details of the uniform distribution and formulae for mean and variance make this question easier.

(c) $\frac{(n+1)(n-1)}{12} = 30$

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Examination style paper
Exercise A, Question 6

Question:

A researcher thinks that there is a link between a person's confidence and their height. She devises a test to measure the confidence, *c*, of nine people and their height, *h* cm. The data are shown in the table below.

h	179	169	187	166	162	193	161	177	168
c	569	561	579	561	540	598	542	565	573

[$\Sigma h = 1562$, $\Sigma c = 5088$, $\Sigma hc = 884\,484$, $\Sigma h^2 = 272094$, $\Sigma c^2 = 2878966$]

a Draw a scatter diagram to represent these data. (2)

b Find the values of S_{hh}, S_{cc} and S_{hc}. (3)

c Calculate the value of the product moment correlation coefficient. (3)

d Calculate the equation of the regression line of c on h. (4)

e Draw this line on your scatter diagram. (2)

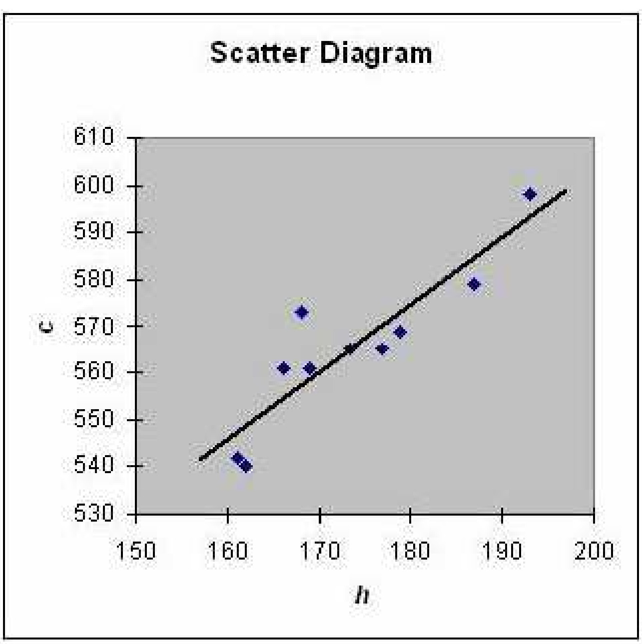
f Interpret the gradient of the regression line. (1)

The researcher decides to use this regression model to predict a person's confidence.

g Find the proposed confidence for the person who has a height of 172 cm. (2)

Solution:

(a) & (e)



Be careful when plotting the points. Make sure the regression line passes through this point. $(\bar{h}, \bar{c})$

B1B1 (2) for points, B1B1 (2) for line.

(b)

$$S_{hh} = 272094 - \frac{1562^2}{9} = 1000.\dot{2}$$

$$S_{cc} = 2878966 - \frac{5088^2}{9} = 2550$$

$$S_{hc} = 884484 - \frac{1562 \times 5088}{9} = 1433.\dot{3}$$

Make sure you work accurately as all these marks are for the answers.

(c)

$$r = \frac{S_{hc}}{\sqrt{S_{hh}S_{cc}}} = \frac{1433.\dot{3}}{\sqrt{1000.\dot{2} \times 2550}} = 0.897488$$

Don't forget the square root.

(d)

$$b = \frac{1433.\dot{3}}{1000.\dot{2}} = 1.433015$$

$$a = \frac{5088}{9} - b \times \frac{1562}{9} = 316.6256$$

$$c = 1.43h + 317$$

Set out your working carefully and remember to subtract the b in the second equation.

(e) See Graph

(f)

For every 1 cm increase in height, the confidence measure increases by 1.43.

This must be in context i.e. it relates to 'height' and 'confidence measure'.

(g)

$$h = 172$$

$$c = 1.43 \times 172 + 317 = 563 \text{ to 3 sf}$$

Substituting h = 172 into your equation gets the method mark.

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Examination style paper
Exercise A, Question 7

Question:

A fairground game involves trying to hit a moving target with an air rifle pellet.

Each player has up to three pellets in a round. Five points are scored if a pellet hits the target, but the round is over if a pellet misses the target.

Jean has a constant probability of 0.4 of hitting the target.

The random variable X is the number of points Jean scores in a round.

Find

a the probability that Jean scores 15 points in a round, (2)
b the probability distribution of X. (5)

A game consists of two rounds.

c Find the probability that Jean scores more points in her second round than her first. (6)

Solution:

(a) P(Scores 15 points)

= P(hit,hit,hit) = $0.4 \times 0.4 \times 0.4 = 0.064$

There is only one way of scoring 15 points.

(b)

x	0	5	10	15
$P(X = x)$	0.6	0.4×0.6	$0.4^2 \times 0.6$	
	0.6	0.24	0.096	0.064

Set out the distribution in a table

(c)

P(Jean scores more in round two than round one)
$= P(X = 0 \text{ then } X = 5, 10 \text{ or } 15)$
$+P(X = 5 \text{ then } X = 10 \text{ or } 15)$
$+P(X = 10 \text{ then } X = 15)$

There is only 1 way of scoring each value as the round ends if Jean misses.

$= 0.6 \times (0.24 + 0.096 + 0.064)$
$+0.24 \times (0.096 + 0.064)$
$+0.096 \times 0.064$

Consider the possible score for the first round in turn and the corresponding scores on the second round.

= 0.284544
= 0.285 (3 sf)